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The Doppler Effect in Expanding Cosmology

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Abstract

In this paper, we study the Doppler effect and derive formulas to calculate it as a function of the relative velocity of a source and observer We then use this to compute the relative velocity of the galaxies with respect to Earth using Hubble's law and the idea of the expanding universe

1 Introduction

When two objects are at rest, the frequency of a light signal emitted by the source Vs is equal to the frequency of the same light signal observed by the observer Vo. But if one of them is moving relative to the other one, then Vs is not equal to Vo. This effect is called the Doppler shift and can be used to tell if an object is moving with respect to another one also the relative velocity can be calculated from the shift. Hubble used this effect to find that all the galaxies are moving away from us at an accelerated rate, meaning the universe is expanding. Moreover, he found that the universe is expanding at an accelerated rate

2 The Doppler Effect

The Doppler effect describes the change in frequency of any sound or light wave produced by a moving source with respect to an observer. When waves travel toward an observer, they get compressed and the frequency increases as the source approaches the observer. [1]

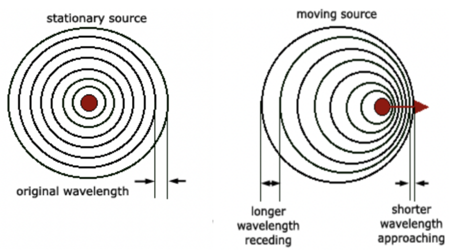

Figure 2.1: A stationary sound source produces sound waves at a constant frequency f, and the wavefronts propagate symmetrically away from the source at a constant speed c. The distance between wavefronts is the wavelength. All observers will hear the same frequency, which will be equal to the actual frequency of the source where

$$f = f_0$$

For a moving source, the same sound source is radiating sound waves at a constant frequency in the same medium. Since the source is moving, the center of each new wavefront is now slightly displaced to the right. As a result, the wavefronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source. An observer in front of the source will hear a higher frequency and an observer behind the source will hear a lower frequency

The formula for the Doppler Effect is described as:

$$f_{\text{observer}} = V + V_{\text{observer}} f/(V - V_{\text{source}}) \quad (1)$$

[1] https://cmtext.indiana.edu/acoustics/chapter1wavelength2.php

$$f_{observer}:$$

is the observed frequency measure in Hz. This is the pitch that the observer will perceive

$$V:$$

The speed of sound traveling in air (343m/s)

$$V_{observer}:$$

velocity of the observer, it is positive if the observer moves towards what makes the sound. It will be negative if the observer moves away from what makes the sound.

$$V_{source}:$$

velocity of the sound, it is positive if the sound source moves towards the observer, it is negative if the sound source moves away from the observer.

$$f_{actual}:$$

frequency of the sound source.
There are different ways of arranging the doppler effects' formula and it depends on whether the observer/source is stationary, moving towards the source, or away from each other. They are rearranged as shown below:

Doppler Shift: $f_{observer} = f_{source} \frac{(V + V_{observer})}{(V - V_{source})}$	Stationary observer	Observer moving towards source	Observer moving away from source
Stationary source	$f_o = f_s$	$f_o = f_s \frac{(V + V_o)}{V}$	$f_o = f_s \frac{(V - V_o)}{V}$
Source moving towards observer	$f_o = f_s \frac{V}{(V - V_s)}$	$f_o = f_s \frac{(V + V_o)}{(V - V_s)}$	$f_o = f_s \frac{(V - V_o)}{(V - V_s)}$
Source moving away from observer	$f_o = f_s \frac{V}{(V + V_s)}$	$f_o = f_s \frac{(V + V_o)}{(V + V_s)}$	$f_o = f_s \frac{(V - V_o)}{(V + V_s)}$

Figure 2.2

There are several ways to derive each formula before being summarized as one equation. One example is when the source(Y) is moving away from the observer(X):
We can use the fact that wavelength is equal to the speed multiplied by the time period and the period is 1/frequency so we can derive the observer frequency.

$$\lambda_o = \lambda_s + \Delta x$$

$$vT_o = vT_s + v_o T_s$$

$$v/f_o = (v + v_s)/f_s$$
$$f_o = f_s(v/(v + v_s$$

Now if the source is coming toward the observer:

$$\lambda_o = \lambda_s - \Delta x$$
$$vT_o = vT_s - v_oT_s$$
$$v/f_o = (v - v_s)/f_s$$
$$f_o = f_s(v/(v - v_s$$

Observer moving toward the source, source stationary:

$$\lambda_o = vT_o + v_oT_o$$
$$vT_s = (v + v_o)T_o$$
$$v(1/f_s) = (v + v_o)1/f_o$$
$$f_o = (f_s((v + v_o)/v$$

Observer moving away from the source, source stationary:

$$\lambda_o = vT_o - v_oT_o$$
$$vT_s = (v - v_o)T_o$$
$$v(1/f_s) = (v - v_o)1/f_o$$
$$f_o = (f_s((v - v_o)/v$$

When these equation are summarized in one equation it looks like this: (The top sign (+ or -) is for either coming towards and the bottom is as they move away).

$$f_{observer} = (f_{source}((V \pm V_{observer})/(V \mp V_{source}$$

3 Redshift

Redshift is the wavelength of light being stretched which is seen as 'shifted' towards the red end of the spectrum. In terms of sources and observers, similar situations occur to sound waves when a source of sound moves relative to an observer. Redshift is an object moving away from us, blueshift is the object moving towards us.
Redshift allows astronomers and astrophysicists to measure distances for the oldest objects and galaxies in our universe (the ones that are further away) Redshift can now be calculated using a simple formula, one based on wavelength, the other on frequency:
Based on wavelength:

$$z = (\lambda_{obsv} - \lambda_{emit})/\lambda_{emit}$$
$$1 + z = \lambda_{obsv}/\lambda_{emit}$$

Based on frequency:

$$z = (f_{emit} - f_{obsv})/f_{obsv}$$
$$1 + z = f_{emit}/f_{obsv}$$

z represents a dimensionless quantity, lambda represents wavelength and f is frequency [2]

[2]https://ned.ipac.caltech.edu/help/zdef.html

4 Hubble's Law

Hubble's law states that a galaxy's velocity is directly proportional to its distance and helps us understand the state of the universe. It proves that the universe is not static or unchanging as there is a clear correlation between distance and velocity.
A way to understand Hubble's law is comprehending the Hubble Plot.

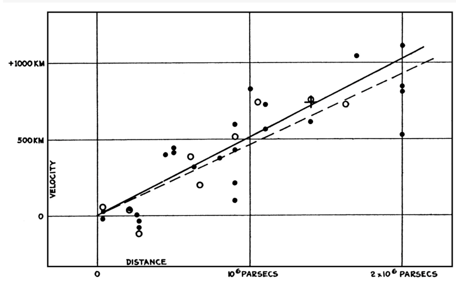

Figure 4.1: This graph plots velocity against distance. The velocity is measured from its spectrum by taking the light from a galaxy's image at the focus of a telescope

[3] Ultimately, Hubble concluded that the redshift of galaxies was directly proportional to the distance of that particular galaxy from Earth. This meant that galaxies further away were moving further away quickly, connoting that the universe is constantly expanding
The conclusion of redshift and Hubble's law is that all galaxies are constantly moving away from us and each other, resulting in the deduction that the universe is constantly expanding. Hubble's law also allows astronomers to calculate how long-ago galaxies started moving apart, which provides information on when the Big Bang occurred- we believe, to our best estimate today, that the Universe is 13.7 billion years old!

5 The Big Bang

The Big Bang explains how the Universe began. It is the idea that the universe started at a single point, and then expanded and stretched and continues to do so today. The Big Bang theory is the most believed state of the Universe as it proves redshift and the fact that the universe is constantly expanding.

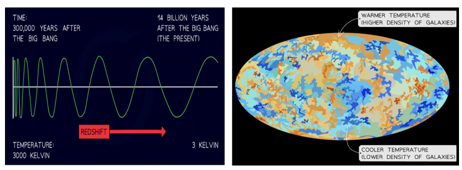

Figure 5.1: The CMB is a result of high energy radiation being redshifted over billions of years

The discovery of Cosmic Microwave Background radiation led to the Big Bang theory becoming what we accept as the model for our universe today. CMBR

[3]https://www.pnas.org/doi/10.1073/pnas.2536799100

is a type of electromagnetic radiation that was found in the early stages of the Universe. CMB is a result of high energy radiation being redshifted over billions of years, it is uniform and follows the profile of being emitted from a hot body that had been cooled down over time. [4]

6 Discussion and Conclusion

Overall, Edwin Hubble has used the Doppler Effect to determine our Universe's expansion. He used redshift to prove light from distant galaxies was shifted toward lower frequencies / the red end of the spectrum. This allows the Big Bang Theory to be proved, as when it occurred, all the matter in the universe moves away from the point of its origin similar to an actual 'bang'. In brief, calculating how fast galaxies are moving away from Earth helps determine the age of our universe as well as when the Big Bang occurred, 13.7 billion years ago. It has evolved from tiny particles to the abundantly structured cosmos of today.

[4] https://science.nasa.gov/astrophysics/focus-areas/what-powered-the-big-bang/